

## Section 3.2: Domain and Range of graphs of functions

### Determine if a graph represents $y$ as a function of $x$ .

We learned how to determine if a relation represents  $y$  as a function of  $x$  in the previous section.

1)  $y$  is NOT a function of  $x$  if there are two or more points with the same  $x$  – *value*, but *different*  $y$  – *values*.

2)  $y$  is a function of  $x$  if each  $x$  has a different  $y$ .

To determine if a graph represents  $y$  as a function of  $x$  can be difficult using what we know so far. This is because points on a graph are not always marked. This can make it hard to find points that have the same  $x$  – *values* with different  $y$  – *values*.

There is technique called the vertical line test that is often used to determine if a graph represents  $y$  as a function of  $x$ .

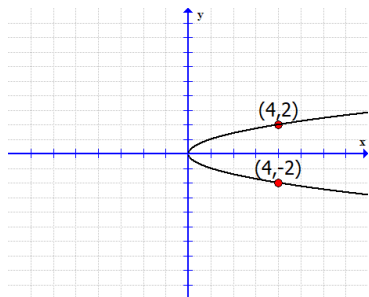
The **vertical line test** is a quick way to determine if a graph represents  $y$  as a function of  $x$  without the need of listing different points with the same  $x$  – *values* but different  $y$  – *values*.

#### Vertical line test:

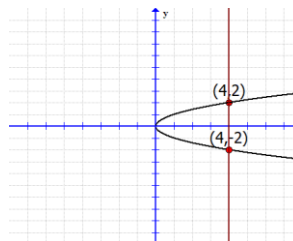
- If a vertical line can be drawn to touch the graph of a function in more than one place, then  $y$  is NOT a function of  $x$ .
- If it is not possible to draw a vertical line to touch the graph of a function in more than one place, then  $y$  is a function of  $x$ .

For Example:

Use the vertical line test to determine if the graph depicts  $y$  is a function of  $x$ .



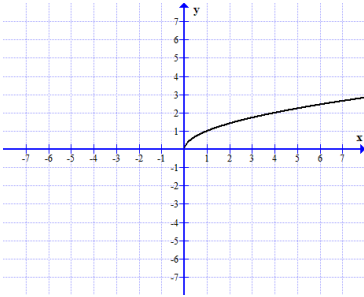
Solution:  **$y$  is NOT a function of  $x$**  (as it is possible to draw a vertical line to touch the graph in more than one place.)



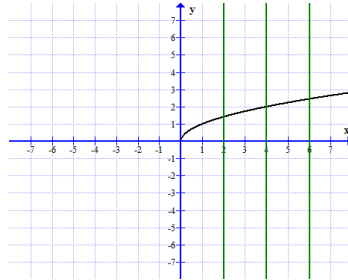
Notice, the vertical line test has found two points on the graph  $\{ (4,2) \text{ and } (4,-2) \}$  that have the same  $x$  with different  $y$ 's.

For Example:

Use the vertical line test to determine if the graph depicts  $y$  is a function of  $x$ .



Solution:  **$y$  is a function of  $x$**  (as no vertical line can be drawn to touch the graph in more than one place.)



## Find the Domain and Range from the graph of a continuous function using INTERVAL NOTATION

Interval notation has two types of symbols:

<b>1) Parenthesis ( )</b> Use round parenthesis when: a) point is marked with an open circle $\circ$ b) value is infinity $\infty$	<b>2) Bracket [ ]</b> Use bracket when: a) point is marked with a closed circle b) point is an unmarked point on a graph
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Writing the domain and or range may require a bracket on one side of the interval and a parenthesis on the other.

- How to find **domain** from a graph of a continuous function and write answer in interval notation.

Step 1) Identify the x-coordinate of left-most the point on the graph. Place it after the appropriate symbol ( or [.

Step 2) Identify the x-coordinate of the right-most point on the graph. Place it before the appropriate symbol ) or ].

- How to find **range** from a graph of a continuous function and write answer in interval notation.

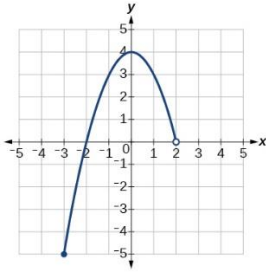
Step 1) Identify the y-coordinate of lowest point on the graph. Place it after the appropriate symbol ( or [.

Step 2) Identify the y-coordinate of the highest point on the graph. Place it before the appropriate symbol ) or ].

It is common for a to graph continue beyond the portion of the graph we can see; the domain and range may be greater than the visible portion of the graph.

We generally need  $\infty$  for one or both sides of the interval of the domain and or range when a graph extends beyond what we can see.

Find the domain and range of the graph below, write answer in interval notation.



Domain:

**First:** Find the x-coordinate of the point that is furthest left and decide whether to put a ( or [ before the number.

The point that is furthest left is (-3, -5). The x-coordinate of the point is  $x = -3$ .

“[” is needed as the point is marked with a closed circle.

The domain will start with [-3,

**Second:** Find the x-coordinate of the point that is furthest right and decide whether to put a ) or ] after it.

The point that is furthest right is (2,0). The x-coordinate is  $x = 2$ .

”)” is needed as the point is marked with an open circle.

The domain will end with ,2)

Answer: Domain [-3,2)

Range:

**First:** Find the y-coordinate of the bottom point and decide whether to put a ( or [ before the number.

The bottom point is (-3,-5). The y-coordinate of the top point is  $y = -5$

“[” is needed as the point is marked with a closed circle.

The range will start with [-5

**Second:** Find the y-coordinate of the top point and decide whether to put a ) or ] after it.

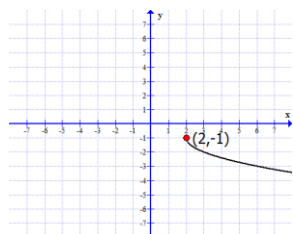
The top point is (0,4). The y-coordinate of the top point is  $y = 4$ .

”)” will be needed to end the range as the point (0,4) is an unmarked point on the graph.

y to end the range: ,4]

Answer: Range [-5, 4]

For Example: Find the domain and range of the graph below, write answer in interval notation.



Domain:

**First:** Find the x-coordinate of the point that is furthest left and decide whether to put a ( or [ before the number.

The point that is furthest left is (2, -1). The x-coordinate of the point is  $x = 2$ .

” [“ is needed as the point is marked with a closed circle.

The domain will start with [2,

**Second:** Find the x-coordinate of the point that is furthest right and decide whether to put a ) or ] after it.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the far-right edge of the x-axis. When a graph extends to the far-right edge of the x-axis,  $\infty$  will be needed to end the domain.

” )” is needed: round parenthesis are always used for  $\infty$ .

The domain will end with  $), \infty)$

Answer: Domain  $[-1, \infty)$

Range:

**First:** Find the y-coordinate of the bottom point and decide whether to put a ( or [ before the number.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the bottom of the y-axis. When a graph extends to the bottom of the y-axis,  $-\infty$  will be needed to start the range.

“(“ is needed: round parenthesis are always used for  $\infty$  and  $-\infty$ .

The range will start with  $(-\infty,$

**Second:** Find the y-coordinate of the top point and decide whether to put a ) or ] after it.

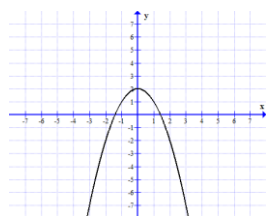
The top point is (2, -1). The y-coordinate of the top point is  $y = -1$ .

” ]” will be needed to end the range as the point (2, -1) is marked with a closed circle.

y to end the range:  $), -1]$

Answer: Range  $(-\infty, -1]$

Find the domain and range of the graph below, write answer in interval notation.



Domain:

**First:** Find the x-coordinate of the point that is furthest left and decide whether to put a ) or ] after it.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the far-left edge of the x-axis. When a graph extends to the far-left edge of the x-axis,  $-\infty$  will be needed to start the domain.

"(" is needed: round parenthesis are always used for  $\infty$ .

The domain will start with  $(-\infty$

**Second:** Find the x-coordinate of the point that is furthest right and decide whether to put a ) or ] after it.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the far-right edge of the x-axis. When a graph extends to the far-right edge of the x-axis,  $\infty$  will be needed to end the domain.

")" is needed: round parenthesis are always used for  $\infty$ .

The domain will end with  $, \infty)$

**Answer: Domain**  $(-\infty, \infty)$

Range:

**First:** Find the y-coordinate of the bottom point and decide whether to put a ( or [ before the number.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the bottom edge of the y-axis on both sides of the graph. When a graph extends to the bottom of the y-axis,  $-\infty$  will be needed to start the range.

"(" is needed: round parenthesis are always used for  $\infty$  and  $-\infty$ .

The range will start with  $(-\infty,$

**Second:** Find the y - coordinate of the top point and decide whether to put a ) or ] after it.

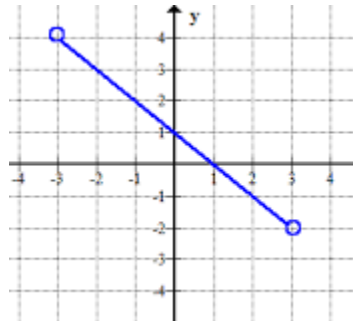
The top point is (0,2). The y - coordinate of the top point is  $y = 2$ .

")" will be needed to end the range as the point (0,2) is unmarked.

y to end the range:  $, 2]$

**Answer: Range**  $(-\infty, 2]$

Find the domain and range of the graph below, write answer in interval notation.



Domain:

**First:** Find the  $x$  – *coordinate* of the point that is furthest left and decide whether to put a ( or [ before the number.

The point that is furthest left is  $(-3, 4)$ . The  $x$  – coordinate of the point is  $x = -3$ .

“(“ is needed as the point is marked with an open circle.

The domain will start with  $(-3,$

**Second:** Find the  $x$  – *coordinate* of the point that is furthest right and decide whether to put a ) or ] after it.

The point that is furthest right is  $(3, -2)$ .  
The  $x$  – *coordinate* is  $x = 3$

The domain will end with  $, 3)$

”)” is needed as the point is marked with an open circle.

Answer: Domain  $(-3, 3)$

Range:

**First:** Find the  $y$  – *coordinate* of the bottom point and decide whether to put a ( or [ before the number.

The bottom point is  $(3, -2)$ . The  $y$  – *coordinate* of the bottom point is  $y = -2$

“(“ is needed as the point is marked with an open circle.

The range will start with  $(-2$

**Second:** Find the  $y$  – *coordinate* of the top point and decide whether to put a ) or ] after it.

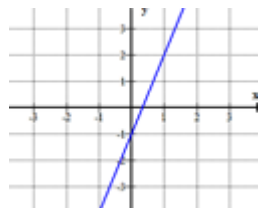
The top point is  $(-3, 4)$ . The  $y$  – *coordinate* of the top point is  $y = 4$ .

”)” will be needed to end the range as the point  $(-3, 4)$  is marked with an open circle.

$y$  to end the range:  $, 4)$

Answer: Range  $(-2, 4)$

Find the domain and range of the graph below, write answer in interval notation.



Domain:

**First:** Find the x-coordinate of the point that is furthest left and decide whether to put a ) or ] after it.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the far-left edge of the x-axis. When a graph extends to the far-left edge of the x-axis,  $-\infty$  will be needed to start the domain.

"(" is needed: round parenthesis are always used for  $\infty$ .

The domain will start with  $(-\infty$

**Second:** Find the x-coordinate of the point that is furthest right and decide whether to put a ) or ] after it.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the far-right edge of the x-axis. When a graph extends to the far-right edge of the x-axis,  $\infty$  will be needed to end the domain.

)" is needed: round parenthesis are always used for  $\infty$ .

The domain will end with  $, \infty)$

Answer: Domain  $(-\infty, \infty)$

Range:

**First:** Find the y-coordinate of the bottom point and decide whether to put a ( or [ before the number.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the bottom edge of the y-axis on both sides of the graph. When a graph extends to the bottom of the y-axis,  $-\infty$  will be needed to start the range.

"(" is needed: round parenthesis are always used for  $\infty$  and  $-\infty$ .

The range will start with  $(-\infty,$

**Second:** Find the y-coordinate of the top point and decide whether to put a ( or [ before the number.

This is a situation where the graph continues beyond what can be seen.

The graph extends to the top edge of the y-axis on both sides of the graph. When a graph extends to the bottom of the y-axis,  $\infty$  will be needed to end the range.

"(" is needed: round parenthesis are always used for  $\infty$  and  $-\infty$ .

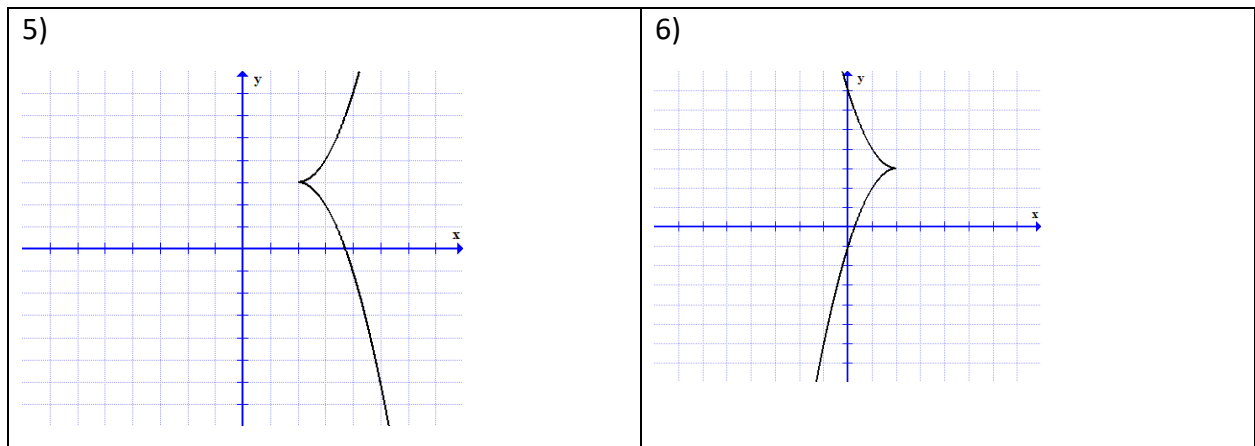
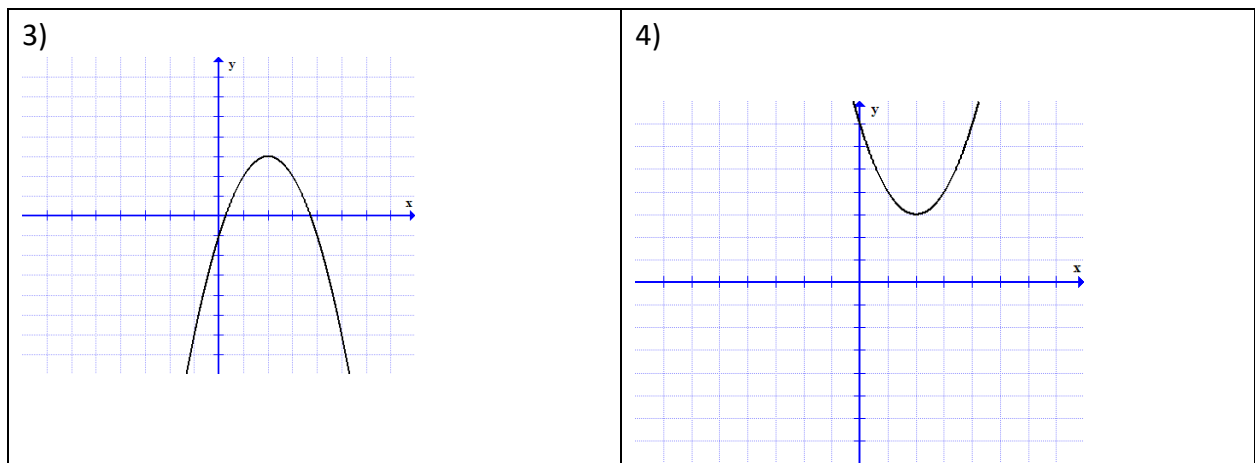
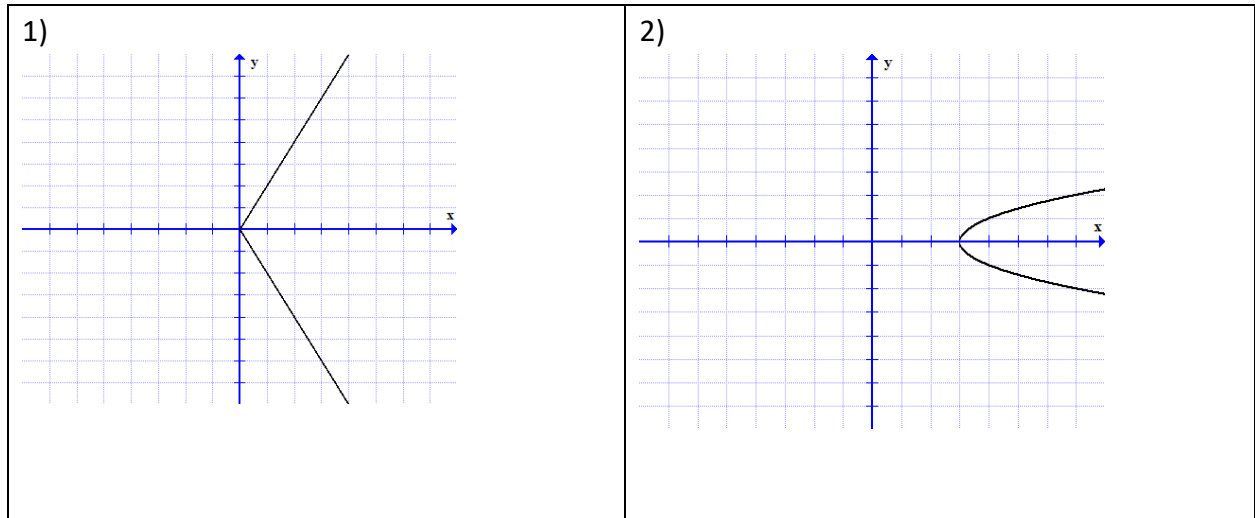
The range will end with  $, \infty)$

Answer: Range  $(-\infty, \infty)$



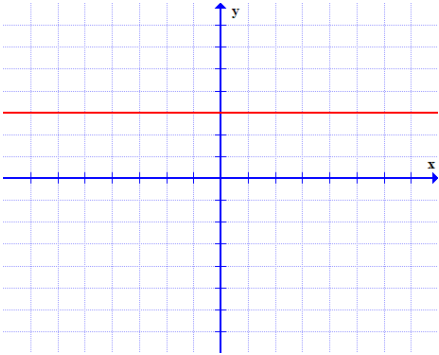
Section 3.2: Domain and Range of graphs of functions

#1-10: Use the vertical line test to determine if the graph represents  $y$  as a function of  $x$ .

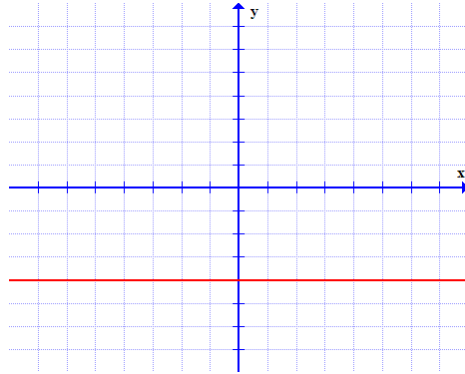


#1-10 Continued: Use the vertical line test to determine if the graph represents  $y$  as a function of  $x$ .

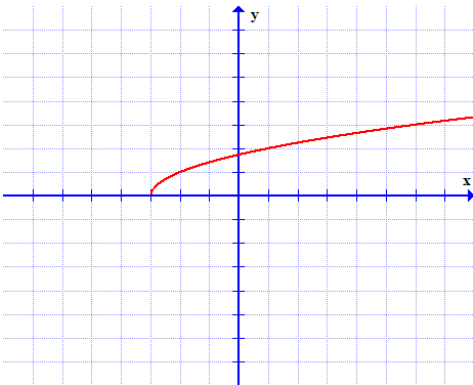
7)



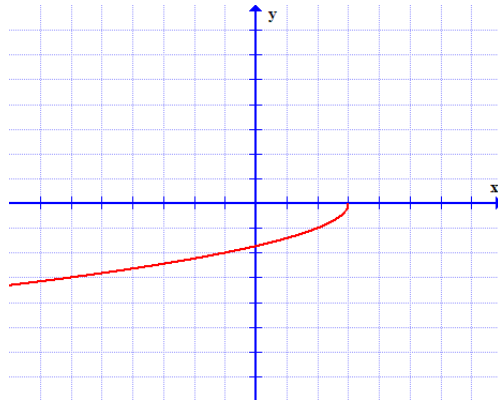
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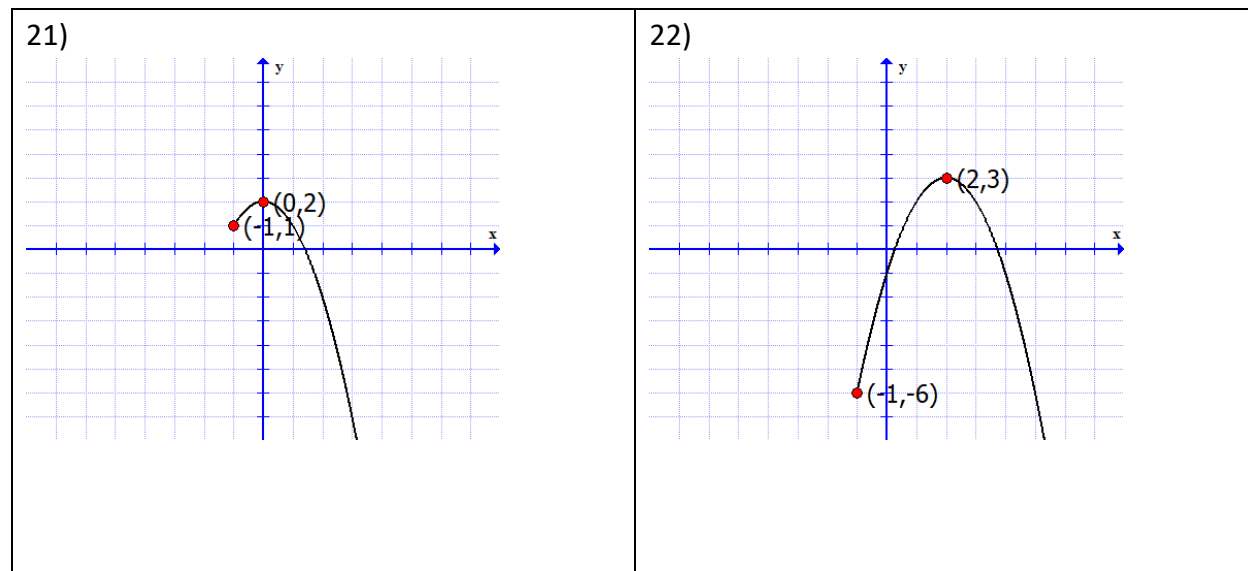
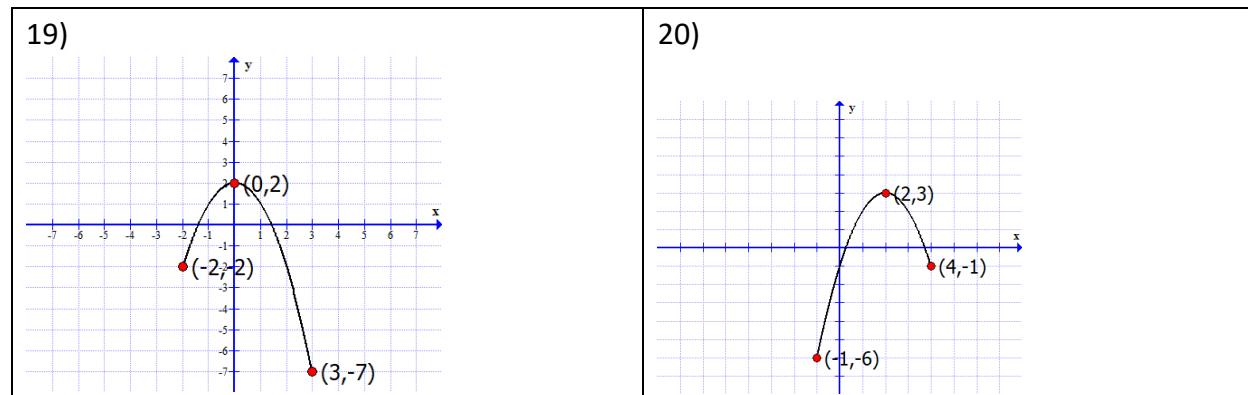
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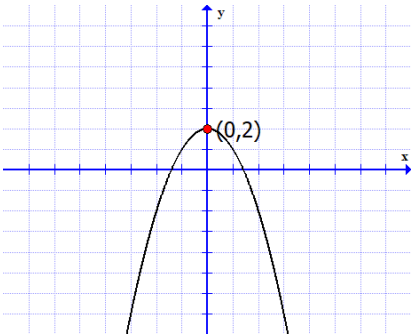
#11 – 2: Determine whether the equation defines  $y$  as a function of  $x$ . Hint, solve the equation for  $y$  and sketch a graph using your calculator, then apply the vertical line test.

- 11)  $y = x^2$                       12)  $y = x^2 + 4$                       13)  $y = \sqrt{x + 2}$                       14)  $y = \sqrt{x - 2}$   
 15)  $y^2 + x^2 = 9$                       16)  $(x-2)^2 + y^2 = 16$                       17)  $x = y^2$                       18)  $x + 2 = y^2$

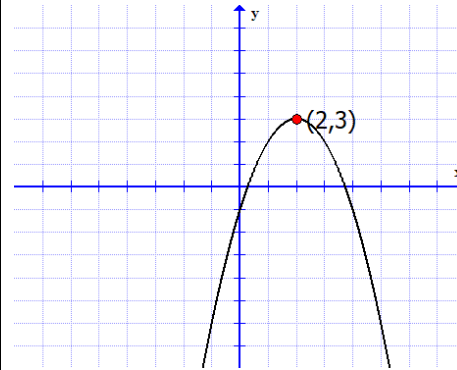
#19 – 38: Find the domain and range of function. Write your answer in interval notation.



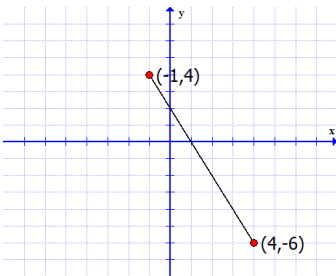
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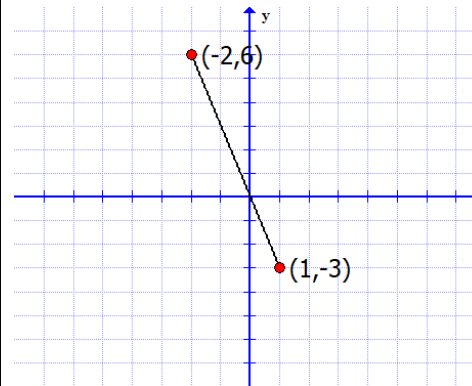
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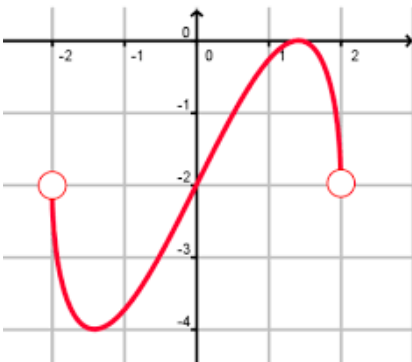
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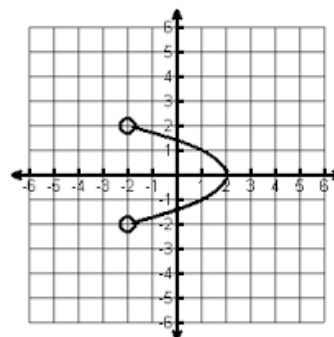
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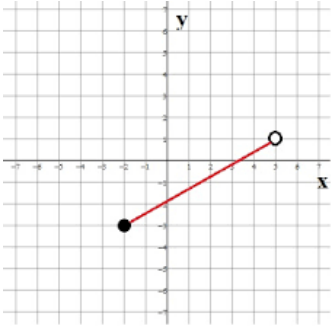
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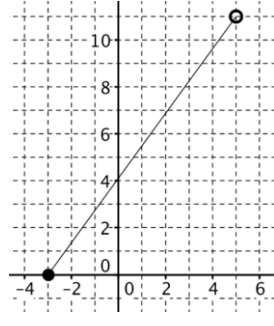
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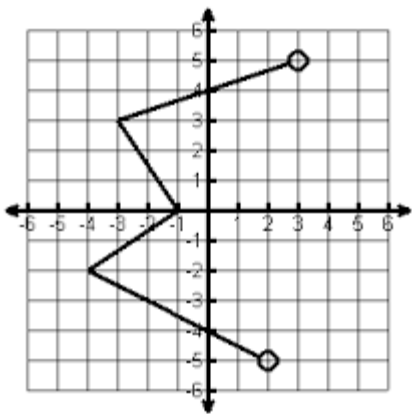
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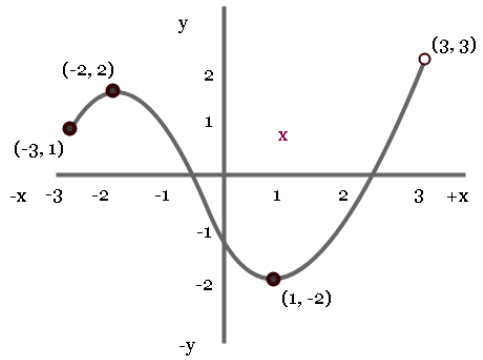
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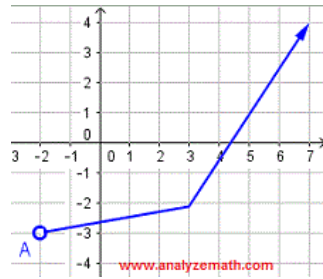
31)



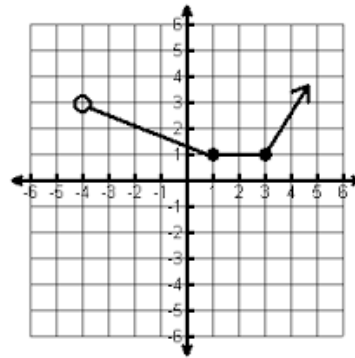
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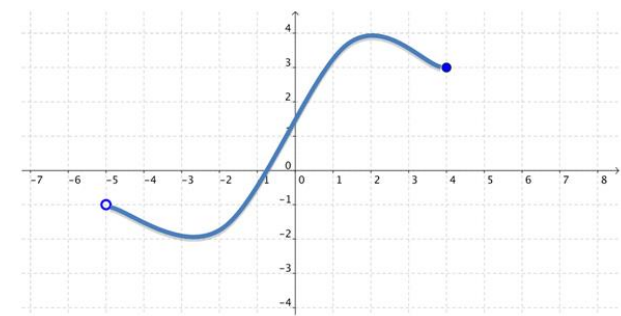
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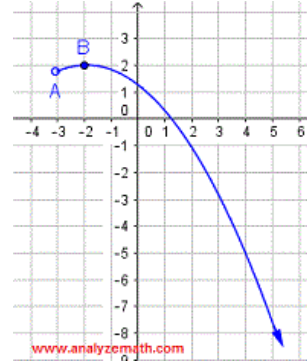
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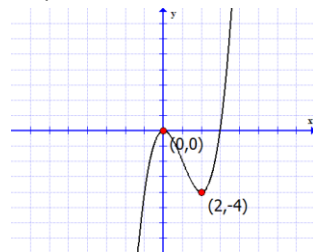
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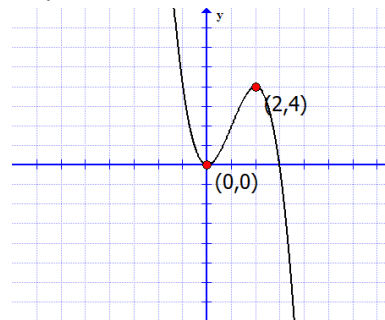
36)



37)



38)



#39 – 53 Use algebra to find the domain of each function. Write your answer in interval notation.

39)  $f(x) = \sqrt{x - 2}$

40)  $f(x) = \sqrt{x - 3}$

41)  $g(x) = \sqrt{3x + 12}$

42)  $g(x) = \sqrt{2x + 10}$

43)  $f(x) = \frac{x+2}{x-3}$

44)  $f(x) = \frac{x-6}{x-7}$

45)  $f(x) = \frac{2}{x^2+6x-7}$

46)  $g(x) = \frac{5}{x^2-5x+6}$

47)  $f(x) = 3x + 6$

48)  $g(x) = 2x - 10$

49)  $f(x) = x^2 + 4$

50)  $g(x) = x^2 + 5$

51)  $h(x) = x^2 + 6x - 7$

52)  $f(x) = x^2 + 2x - 15$